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Attitude Control of Space Robot by Arm Motion

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A path-planning method of a space robot manipulator arm is presented in this paper. The purpose of the path planning is to control the satellite's attitude by the arm's motion. The joint angles are expressed by two parameters and a relation between the attitude change and the motion of the parameters is derived. Based on this relation, an algorithm for the joint angle path to cause the desired attitude change is proposed. An attitude control measure is also introduced to show the difficulty of changing attitude by arm motion. Numerical studies are executed using a space robot model with a manipulator arm with six degrees of freedom. The result shows that there are some directions where the attitude change is easily achieved by the arm motion. The validity of the proposed algorithm is confirmed by numerical simulations.

Nomenclature

I_3 = 3 × 3 identity matrix
$\hat{I}_i$ = inertia tensor of outward bodies $\hat{i}$ about their CM
$\hat{m}_i$ = mass of outward bodies $\hat{i}$
p_i = position vector of joint i
$\hat{p}_i$ = position vector of center of mass (CM) of outward bodies $\hat{i}$
z_i = unit vector along rotational axis of joint i
ϵ = scalar part of Euler parameters
$\boldsymbol{\epsilon}$ = vector part of Euler parameters
θ_i = relative rotation angle of joint i
ω = angular velocity of satellite

Introduction

ROBOTS working in space have been actively studied in recent years.[1–4] A free-flying space robot differs from a robot fixed on the ground because the satellite body is moved by the motion of the manipulator arm. In this paper, we consider the possibility of the attitude control of a space robot by arm motion. The method is to utilize the attitude change when the arm moves from the initial posture to the final posture along a certain path. If the attitude change due to the arm motion can be compensated by a well-designed path for the arm, the attitude control system of space robots can be simplified. For example, small space robots do not always have reaction wheels for attitude control. Even then an effective attitude control system can be constructed by combining thrusters and a manipulator arm.

The physical features of this kind of attitude control have been investigated as the "falling-cat phenomenon"[5] or as the attitude change of astronauts.[6] Also some algorithms have already been proposed for attitude control. Reyhanoglu and McClamroch[7] and Krishnaprasad[8] proposed a method for planar attitude change using the joint motion. Vafa and Dubowsky[9] and Longman[10] proposed methods of three-dimensional attitude control using the cyclic motions of the arm. However, a solution for a general arm with n degrees of freedom has not been obtained in both methods. Nakamura and Mukherjee[11] proposed a method using a Lyapunov function called the bidirectional approach. This method can control the satellite attitude and the arm joint angles simultaneously, but the optimality of the arm motion is not considered.

The method proposed here is to prescribe the arm path first and find the quasi-optimal path that causes the desired attitude change with the minimum arm movement. The method can easily find the path for a space robot with an arm of n degrees of freedom in cases where the arm's initial and final postures are the same. A measure that shows the difficulty of attitude change by arm motion is also introduced. Numerical studies are executed to check the measure for all directions of the attitude change and to verify the obtained path.

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Expression of Attitude Change Caused by Arm Motion

First the attitude change of the satellite (main body of the space robot) when the arm moves from an initial posture to the same final posture is considered. The basic relations of attitude change by arm motion are derived when the arm motion is infinitely small. Let us consider here the case where the space robot has one manipulator arm with n rotational degrees of freedom and the momentum and the angular momentum of the space robot are conserved at 0 during the arm motion. The satellite body is numbered 0 and the bodies of the arm are numbered $1, \ldots, n$ from the shoulder to the hand. We define outward bodies $\hat{i}$ as a set of bodies $i, i+1, \ldots, n$ and also define joint i as a joint that connects body i and body $i-1$. Other symbols are defined in the Nomenclature. In this paper, all vectors and tensors are expressed in inertial coordinates.

As the angular momentum of the space robot about its center of mass (CM) is zero, the following equation holds[12]:

$$\hat{I}_0\omega + \sum_{i=1}^{n} \tilde{I}_i z_i \dot{\theta}_i = \mathbf{0} \tag{1}$$

$$\tilde{I}_i = \hat{I}_i + \hat{m}_i\left[(\hat{p}_i - p_i)\cdot(\hat{p}_i - \hat{p}_0)I_3 - (\hat{p}_i - p_i)(\hat{p}_i - \hat{p}_0)^T\right] \tag{2}$$

Let us define vector x_i as

$$\hat{I}_0 x_i = \tilde{I}_i z_i \tag{3}$$

Then the angular velocity ω is expressed as

$$\omega = -\sum_{i=1}^{n} x_i \dot{\theta}_i \tag{4}$$

The vector part of Euler parameters $\boldsymbol{\epsilon}$ is used as the satellite attitude. The time derivative of $\boldsymbol{\epsilon}$ in the inertial coordinates has the following relation with ω:

$$\dot{\boldsymbol{\epsilon}} = \boldsymbol{\epsilon} \circ \omega \tag{5}$$

where $\circ$ denotes the operator between $\boldsymbol{\epsilon}$ and any 3 × 1 vector $\boldsymbol{v}$, giving

$$\boldsymbol{\epsilon} \circ \boldsymbol{v} = \tfrac{1}{2}\epsilon \boldsymbol{v} - \tfrac{1}{2}\boldsymbol{\epsilon} \times \boldsymbol{v} \tag{6}$$

When each joint of the arm moves from an initial posture to the same posture along a path, we can express the attitude change Δ_ϵ

(change of ϵ) as

$$\boldsymbol{\Delta}_\epsilon = -\sum_{i=1}^{n} \int \boldsymbol{\epsilon} \circ \boldsymbol{x}_i \, d\theta_i \tag{7}$$

where the integration is linear integral along the joint path.

For simplicity, each joint angle θ_i will be expressed by two parameters p_1 and p_2:

$$\theta_i = \theta_i(p_1, p_2) \tag{8}$$

The reason for two parameters is that the minimum number is 2. As described later, an attitude change does not occur if only one parameter is used.

Using the expression of joint angles, the path in the $p_1 p_2$ parameter plane becomes a closed curve. Hereafter the differential forms for mathematical treatment will be used. Let us define the region bounded by the closed curve as C and its boundary as ∂C. When the arm motion is infinitely small, the attitude change is expressed by the theorem of exterior derivative[13] as

$$\boldsymbol{\Delta}_\epsilon = -\sum_{i=1}^{n} \int_{\partial C} \boldsymbol{\sigma}_i = -\sum_{i=1}^{n} \int_{C} d\boldsymbol{\sigma}_i \tag{9}$$

where $\boldsymbol{\sigma}_i$ is the following differential 1-form

$$\boldsymbol{\sigma}_i = \boldsymbol{\epsilon} \circ \boldsymbol{x}_i \, d\theta_i \tag{10}$$

and d denotes the exterior derivative. To calculate $d\boldsymbol{\sigma}_i$ in Eq. (9), the following relations for the solutions of Eqs. (4) and (5) are used:

$$d\boldsymbol{x}_i = \sum_{j=1}^{n} \left(\frac{\partial \boldsymbol{x}_i}{\partial \theta_j} + \boldsymbol{x}_i \times \boldsymbol{x}_j \right) d\theta_j, \qquad d\boldsymbol{\epsilon} = -\boldsymbol{\epsilon} \circ \sum_{j=1}^{n} \boldsymbol{x}_j \, d\theta_j, \tag{11}$$

$$d\epsilon = \frac{1}{2} \boldsymbol{\epsilon} \cdot \sum_{j=1}^{n} \boldsymbol{x}_j \, d\theta_j$$

The first equation of Eq. (11) is obtained from the time derivative of the vector $\boldsymbol{x}_i$ in the inertial coordinates:

$$\dot{\boldsymbol{x}}_i = \sum_{j=1}^{n} \frac{\partial \boldsymbol{x}_i}{\partial \theta_j} \dot{\theta}_j + \boldsymbol{\omega} \times \boldsymbol{x}_i \tag{12}$$

The term $\partial \boldsymbol{x}_i / \partial \theta_j$ means the partial differential of $\boldsymbol{x}_i$ by θ_j when θ_j and $\boldsymbol{\epsilon}$ are regarded as independent.

Substituting Eq. (11) into Eq. (9) yields

$$\Delta_\epsilon = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} \int_{C} \boldsymbol{v}_{ij} \, d\theta_i \wedge d\theta_j \tag{13}$$

$$\boldsymbol{v}_{ij} = \boldsymbol{\epsilon} \circ \left(\frac{\partial \boldsymbol{x}_i}{\partial \theta_j} - \frac{\partial \boldsymbol{x}_j}{\partial \theta_i} + \boldsymbol{x}_i \times \boldsymbol{x}_j \right) \tag{14}$$

where $d\theta_i \wedge d\theta_j$ is a differential 2-form and the following identities are used:

$$d\theta_i \wedge d\theta_j = -d\theta_j \wedge d\theta_i,$$
$$\sum_{i=1}^{n} \sum_{j=1}^{n} \boldsymbol{v}_{ij} \, d\theta_i \wedge d\theta_j = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} (\boldsymbol{v}_{ij} - \boldsymbol{v}_{ji}) \, d\theta_i \wedge d\theta_j \tag{15}$$

From Eq. (8), $d\theta_i \wedge d\theta_j$ means

$$d\theta_i \wedge d\theta_j = \left(\frac{\partial \theta_i}{\partial p_1} \frac{\partial \theta_j}{\partial p_2} - \frac{\partial \theta_i}{\partial p_2} \frac{\partial \theta_j}{\partial p_1} \right) dp_1 \wedge dp_2 \tag{16}$$

where $dp_1 \wedge dp_2$ is an areal element in the $p_1 p_2$ parameter plane. Here let us express the joint angle θ_i by its simplest form as

$$\theta_i = a_i p_1 + b_i p_2 + c_i \tag{17}$$

where a_i, b_i, and c_i are constant parameters. Substituting Eqs. (16) and (17) into Eq. (13), we can express $\boldsymbol{\Delta}_\epsilon$ as

$$\boldsymbol{\Delta}_\epsilon = \begin{bmatrix} \boldsymbol{a} \cdot \boldsymbol{D}_1 \boldsymbol{b} \\ \boldsymbol{a} \cdot \boldsymbol{D}_2 \boldsymbol{b} \\ \boldsymbol{a} \cdot \boldsymbol{D}_3 \boldsymbol{b} \end{bmatrix} \tag{18}$$

where

$$\boldsymbol{a} = [\, a_1 \, a_2 \cdots a_n \,]^T, \qquad \boldsymbol{b} = [\, b_1 \, b_2 \cdots b_n \,]^T \tag{19}$$

and $\boldsymbol{D}_1, \boldsymbol{D}_2$ and $\boldsymbol{D}_3$, are matrices the $(i,\ j)$ components of which are

$$\begin{bmatrix} \boldsymbol{D}_{1ij} \\ \boldsymbol{D}_{2ij} \\ \boldsymbol{D}_{3ij} \end{bmatrix} = \boldsymbol{v}_{ij} \int_{C} dp_1 \wedge dp_2 \tag{20}$$

As Eq. (9) holds true for infinitely small C, $\boldsymbol{v}_{ij}$ is put out of the integral in Eq. (20).

Equation (18) means that attitude of a space robot becomes $\boldsymbol{\epsilon} + \Delta_\epsilon$ from $\boldsymbol{\epsilon}$ (initial attitude) when the arm moves along the small boundary ∂C in the $p_1 p_2$ parameter plane. The attitude change $\boldsymbol{\Delta}_\epsilon$ is proportional to the area integration of C in the parameter plane. Though Eq. (18) holds for small arm motions, this equation can be applied to large arm motions to a first approximation.

Arm Path Solution

Approximate Solution

The problem is to find the arm path that makes the attitude change $\boldsymbol{\Delta}_\epsilon$ the desired value $\boldsymbol{\Delta}_d$, where the arm's final posture is the same as the initial one. The solution will be obtained by the following two steps:

1) Based on Eq. (18), we will first find the solution ($\boldsymbol{a}$ and $\boldsymbol{b}$) that satisfies $\boldsymbol{\Delta}_\epsilon = \boldsymbol{\Delta}_d$. The matrices $\boldsymbol{D}_1$, $\boldsymbol{D}_2$, and $\boldsymbol{D}_3$ are evaluated by $\boldsymbol{v}_{ij}$ at the initial state of the space robot and the area of the region C in the parameter plane. However, the constraint $\boldsymbol{\Delta}_\epsilon = \boldsymbol{\Delta}_d$ is not really satisfied by this solution, because Eq. (18) holds for small arm motions. This solution is called an approximate solution.

2) Based on Eq. (7), the approximate solution is modified to satisfy $\boldsymbol{\Delta}_\epsilon = \boldsymbol{\Delta}_d$ exactly. This solution is a modified solution.

In this subsection the method of obtaining the approximate solution is described. The method of modifying the approximate solution is given in the next subsection.

The approximate solution is obtained by finding the coefficient vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ that satisfy $\boldsymbol{\Delta}_\epsilon$ [Eq. (18)] $= \boldsymbol{\Delta}_d$. There are many combinations of $\boldsymbol{a}$ and $\boldsymbol{b}$ because $\boldsymbol{\Delta}_d$ has only three components. The problem of finding values for $\boldsymbol{a}$ and $\boldsymbol{b}$ that minimize $\boldsymbol{a} \cdot \boldsymbol{a} + \boldsymbol{b} \cdot \boldsymbol{b}$ is considered here. As $\boldsymbol{a}$ and $\boldsymbol{b}$ are the coefficient vectors of parameters p_1 and p_2, the minimization of $\boldsymbol{a} \cdot \boldsymbol{a} + \boldsymbol{b} \cdot \boldsymbol{b}$ means controlling the satellite's attitude with the least joint movement. The problem becomes static optimization with an equality constraint $\boldsymbol{\Delta}_\epsilon = \boldsymbol{\Delta}_d$. Let the multiplier vector $\boldsymbol{\lambda} = [\, \lambda_1 \, \lambda_2 \, \lambda_3 \,]^T$, and the cost function be

$$J = \boldsymbol{a} \cdot \boldsymbol{a} + \boldsymbol{b} \cdot \boldsymbol{b} + \boldsymbol{\lambda} \cdot (\boldsymbol{\Delta}_\epsilon - \boldsymbol{\Delta}_d) \tag{21}$$

Using Eq. (18), the necessary conditions for optimality are obtained by differentiating J with respect to $\boldsymbol{a}$ and $\boldsymbol{b}$ as

$$2\boldsymbol{a} + \boldsymbol{D}_\lambda \boldsymbol{b} = \boldsymbol{0}, \qquad 2\boldsymbol{b} - \boldsymbol{D}_\lambda \boldsymbol{a} = \boldsymbol{0},$$
$$\boldsymbol{D}_\lambda = \lambda_1 \boldsymbol{D}_1 + \lambda_2 \boldsymbol{D}_2 + \lambda_3 \boldsymbol{D}_3 \tag{22}$$

From Eq. (22), we can transform the constraint $\boldsymbol{\Delta}_\epsilon = \boldsymbol{\Delta}_d$ into a linear form of $\boldsymbol{\lambda}$ as

$$\boldsymbol{A}^T \boldsymbol{A} \boldsymbol{\lambda} = -2\boldsymbol{\Delta}_d, \qquad \boldsymbol{A} = [\, \boldsymbol{D}_1 \boldsymbol{a} \quad \boldsymbol{D}_2 \boldsymbol{a} \quad \boldsymbol{D}_3 \boldsymbol{a} \,] \tag{23}$$

Thus $\boldsymbol{\lambda}$ is obtained from Eq. (23) when $\boldsymbol{a}$ is given. On the other hand, we can obtain this relation from Eq. (22),

$$\boldsymbol{a} = -\tfrac{1}{2} \boldsymbol{D}_\lambda \boldsymbol{b} = -\tfrac{1}{4} \boldsymbol{D}_\lambda^2 \boldsymbol{a} \tag{24}$$

Equations (23) and (24) consist of nonlinear equations of the vector $\boldsymbol{a}$. From Eqs. (22) and (23), the solution of these equations has the following characteristics:

$$\boldsymbol{a} \cdot \boldsymbol{b} = 0, \qquad \boldsymbol{a} \cdot \boldsymbol{a} = \boldsymbol{b} \cdot \boldsymbol{b} = -\tfrac{1}{2} \boldsymbol{\lambda} \cdot \boldsymbol{\Delta}_d \tag{25}$$

Thus the cost function J becomes

$$J = -\boldsymbol{\lambda} \cdot \boldsymbol{\Delta}_d \tag{26}$$

As Eqs. (23) and (24) are highly nonlinear for the vector $\boldsymbol{a}$, a numerical method for a nonlinear equation must be used to obtain the solution. Here it is described how to give an appropriate initial value of the vector $\boldsymbol{a}$ for the numerical method and how to verify the optimality of the obtained solution. Then a measure to judge the difficulty of the attitude change will be introduced. First define the matrix $\hat{\boldsymbol{D}}$ the (i, j) components of which are

$$\hat{D}_{ij} = \begin{bmatrix} \boldsymbol{D}_{1ij} \\ \boldsymbol{D}_{2ij} \\ \boldsymbol{D}_{3ij} \end{bmatrix} \cdot \hat{\boldsymbol{\Delta}}_d \tag{27}$$

where $\hat{\boldsymbol{\Delta}}_d = \boldsymbol{\Delta}_d / \|\boldsymbol{\Delta}_d\|$ ($\|\cdot\|$ indicates a Euclidean norm). Considering the component of $\boldsymbol{\Delta}_d$ along the rotational axis and taking an inner product of both sides of $\boldsymbol{\Delta}_\epsilon = \boldsymbol{\Delta}_d$ with $\hat{\boldsymbol{\Delta}}_d$, we obtain the equation

$$\boldsymbol{a} \cdot \hat{\boldsymbol{D}} \boldsymbol{b} = \|\boldsymbol{\Delta}_d\| \tag{28}$$

As $\hat{\boldsymbol{D}}$ is skew symmetric, its eigenvalues are pure imaginary and it is transformed into a diagonal matrix by a unitary matrix. Let the maximum eigenvalues (in absolute value) of $\hat{\boldsymbol{D}}$ be $\pm i\alpha$ ($\alpha > 0$) and the corresponding eigenvectors be $\boldsymbol{u} \pm i\boldsymbol{v}$ ($\|\boldsymbol{u}\| = \|\boldsymbol{v}\| = 1/\sqrt{2}$). If we set $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$ as

$$\hat{\boldsymbol{a}} = \sqrt{\frac{2\|\boldsymbol{\Delta}_d\|}{\alpha}}\boldsymbol{u}, \qquad \hat{\boldsymbol{b}} = \sqrt{\frac{2\|\boldsymbol{\Delta}_d\|}{\alpha}}\boldsymbol{v} \tag{29}$$

and substitute $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$ into $\boldsymbol{a}$ and $\boldsymbol{b}$, respectively, Eq. (28) is satisfied. As α corresponds to the maximum eigenvalue of $\hat{\boldsymbol{D}}$, $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$ are the solutions with a minimum norm. However, the eigenvectors of $\hat{\boldsymbol{D}}$ corresponding to $\pm i\alpha$ are not unique. The eigenvectors $(\cos\gamma + i\sin\gamma)(\boldsymbol{u} \pm i\boldsymbol{v})$ are also normalized eigenvectors. Thus, vectors

$$\boldsymbol{a}_0 = \cos\gamma\hat{\boldsymbol{a}} - \sin\gamma\hat{\boldsymbol{b}}, \qquad \boldsymbol{b}_0 = \sin\gamma\hat{\boldsymbol{a}} + \cos\gamma\hat{\boldsymbol{b}} \tag{30}$$

are thought to be appropriate for the initial values of $\boldsymbol{a}$ and $\boldsymbol{b}$. However, the obtained solution does not necessarily minimize J, because Eqs. (23) and (24) are the only necessary conditions for optimality. Here let us use the following approximate condition to check the optimality. As is clear from Eq. (24), $\boldsymbol{D}_\lambda$ has an eigenvalue of $\pm 2i$ if Eqs. (23) and (24) hold. On the other hand, from Eq. (26), J is thought to be small if $\boldsymbol{\lambda}$ is small. Therefore we can adopt it as an approximate criterion of the optimality that the maximum eigenvalue of $\boldsymbol{D}_\lambda$ is $\pm 2i$. If $\bar{\boldsymbol{a}}$ and $\bar{\boldsymbol{b}}(=\frac{1}{2}\boldsymbol{D}_\lambda\bar{\boldsymbol{a}})$ are obtained to satisfy this criterion, the general form of the solution of $\boldsymbol{a}$ and $\boldsymbol{b}$ is expressed using an arbitrary angle δ as

$$\boldsymbol{a} = (\cos\delta\bar{\boldsymbol{a}} - \sin\delta\bar{\boldsymbol{b}}), \qquad \boldsymbol{b} = (\sin\delta\bar{\boldsymbol{a}} + \cos\delta\bar{\boldsymbol{b}}) \tag{31}$$

The algorithm to obtain the optimal solution is summarized as follows:

1) Calculate $\hat{\boldsymbol{D}}$ from Eq. (27) and obtain $\hat{\boldsymbol{a}}$ and $\hat{\boldsymbol{b}}$ of Eq. (29) from the eigenvector of $\hat{\boldsymbol{D}}$ corresponding to the maximum eigenvalue.

2) Set $\gamma = 0$.

3) Solve nonlinear equations (23) and (24) with $\boldsymbol{a}_0$ (initial value of $\boldsymbol{a}$) $= \cos\gamma\hat{\boldsymbol{a}} - \sin\gamma\hat{\boldsymbol{b}}$. The multiplier vector $\boldsymbol{\lambda}$ is also obtained in this step.

4) If the maximum eigenvalue of $\boldsymbol{D}_\lambda$ is $\pm 2i$, adopt the solution as optimal. The general solution is expressed by Eq. (31). Otherwise, return to step 3 with $\gamma = \gamma + \Delta\gamma$.

If we calculate the solution for various directions keeping $\|\boldsymbol{\Delta}_d\|$ constant, we can regard $\|\boldsymbol{a}\|$ as a measure of the difficulty of the attitude control. That is, the following σ is adopted as the attitude control measure for a certain direction:

$$\sigma = \|\boldsymbol{a}\| = \sqrt{-\tfrac{1}{2}\boldsymbol{\lambda} \cdot \boldsymbol{\Delta}_d} \tag{32}$$

Modified Solution

The constraint condition $\boldsymbol{\Delta}_\epsilon = \boldsymbol{\Delta}_d$ is not usually satisfied by the above approximate solution, because Eq. (18) holds true for infinitely small arm motion. Here the method of obtaining values of $\boldsymbol{a}$ and $\boldsymbol{b}$ that satisfy the constraint condition exactly from the approximate solution will be briefly described. Using the Newton method, $\boldsymbol{a}_i$ and $\boldsymbol{b}_i$ (the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ at the ith iteration) are calculated from $\boldsymbol{a}_{i-1}$ and $\boldsymbol{b}_{i-1}$ as

$$\begin{bmatrix} \boldsymbol{a}_i \\ \boldsymbol{b}_i \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_{i-1} \\ \boldsymbol{b}_{i-1} \end{bmatrix} - \begin{bmatrix} \dfrac{\partial\boldsymbol{\Delta}_\epsilon}{\partial\boldsymbol{a}} & \dfrac{\partial\boldsymbol{\Delta}_\epsilon}{\partial\boldsymbol{b}} \end{bmatrix}^+ (\boldsymbol{\Delta}_\epsilon - \boldsymbol{\Delta}_d) \tag{33}$$

where the superscript plus represents a pseudo-inverse matrix. We can obtain $\boldsymbol{\Delta}_\epsilon$ of Eq. (33) by the numerical integration of Eq. (7) and obtain $\partial\boldsymbol{\Delta}_\epsilon/\partial\boldsymbol{a}$ and $\partial\boldsymbol{\Delta}_\epsilon/\partial\boldsymbol{b}$ by the numerical differentiation of $\boldsymbol{\Delta}_\epsilon$. It takes some computation time to calculate these terms because of the numerical integration. However, the convergence of the Newton method is good and the number of iterations are few as the approximate solution has already been obtained. We can obtain from Eq. (33) the quasi-optimal solution that satisfies the constraint condition.

Numerical Study

Using the algorithm described above, numerical studies of the attitude control have been made. Figure 1 shows a space robot model. The space robot has an arm with six degrees of freedom. The arm is attached to the satellite body at coordinates [0, 1.0, 0.5] (meters) from the satellite CM. The main parameters are shown in Table 1. Let us set the initial joint angles of the arm as [0 deg, 60 deg, −60 deg, 0 deg, −30 deg, 0 deg] from θ_1 to θ_6 where all joint angles are 0 deg at the posture shown in Fig. 1. Also set the attitude change of the space robot as the rotation of angle ψ around the unit vector $\boldsymbol{e}$ defined by ϕ and ϑ in Fig. 2.

Figure 3 shows the attitude control measure σ for all directions of $\boldsymbol{e}$ when $\boldsymbol{\Delta}_d = \boldsymbol{e}$. This figure plots the point of $\sigma\boldsymbol{e}$ where $\boldsymbol{e}$ varies at intervals of 10 deg from 0 deg up to 360 deg for ϕ and at intervals of 5 deg from −90 deg up to 90 deg for ϑ. We can calculate the attitude control measure for every directionof $\boldsymbol{e}$ by the proposed algorithm. A pole of Fig. 3 corresponds to the point $\vartheta = 90$ deg. In this case, the maximum of σ ($\sigma_{\max}$) and the minimum of σ ($\sigma_{\min}$) become as follows:

$$\begin{aligned} \sigma_{\max} = 5.00&: \phi = 90\text{ deg}, \vartheta = -60\text{ deg}; \\ &\phi = 270\text{ deg}, \vartheta = 60\text{ deg} \\ \sigma_{\min} = 1.14&: \phi = 90\text{ deg}, \vartheta = 30\text{ deg (dent of Fig. 3)}; \\ &\phi = 270\text{ deg}, \vartheta = -30\text{ deg} \end{aligned} \tag{34}$$

Table 1 Space robot parameters

		Manipulator arm					
	Satellite	L1	L2	L3	L4	L5	L6
Length, m	ϕ2.4 × 1	0.2	1	0.5	0.5	0.1	0.2
Mass, kg	500	10	30	20	20	10	20
Moments	100	0.1	0.2	0.2	0.2	0.1	0.1
of Inertia,	100	0.1	3	0.5	0.5	0.1	0.1
kgm^2	100	0.1	3	0.5	0.5	0.1	0.1

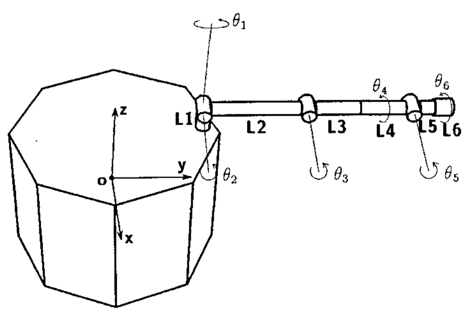

Fig. 1 Space robot model.

Table 2 Components of *a* and *b* for $\sigma = \sigma_{\max}$ and $\sigma = \sigma_{\min}$

σ	Solution		Components of *a* and *b* 1	2	3	4	5	6
$\sigma_{\max}$	Approximate	*a*	1.016	−0.563	0.877	0.048	0.243	0.004
		b	1.069	0.535	−0.834	0.051	−0.231	0.004
	Modified	*a*	0.947	−0.504	0.852	0.045	0.234	0.004
		b	0.999	0.491	−0.806	0.047	−0.222	0.004
$\sigma_{\min}$	Approximate	*a*	0.243	0.186	0.137	0.011	0.012	−0.000
		b	0.231	−0.195	−0.144	0.010	−0.013	−0.000
	Modified	*a*	0.248	0.198	0.126	0.010	0.009	−0.000
		b	0.236	−0.203	−0.140	0.009	−0.011	−0.000

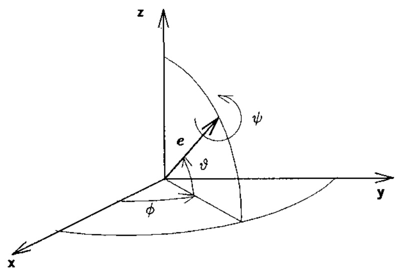

Fig. 2 Rotational axis of the space robot.

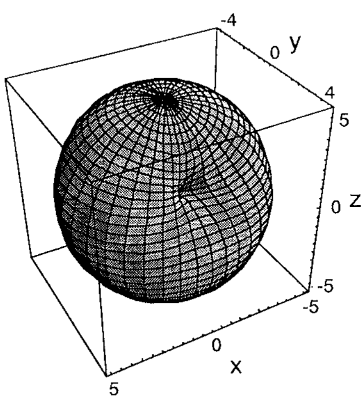

Fig. 3 Attitude control measure.

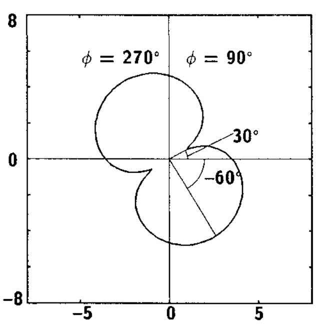

Fig. 4 Cross section of Fig. 3 at $\sigma = \sigma_{\max}$ and $\sigma = \sigma_{\min}$.

Table 3 Attitude change Δ_ϵ for $\sigma = \sigma_{\max}$ and $\sigma = \sigma_{\min}$

	Solution	Attitude change Δ_ϵ x	y	z
$\sigma_{\max}$	Approximate	0.0982	0.0003	−0.0453
	Modified	0.0872	0	0
$\sigma_{\min}$	Approximate	0.0839	−0.0001	−0.0002
	Modified	0.0872	0	0

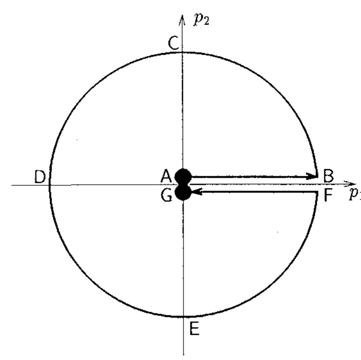

Fig. 5 Path in the p_1p_2 parameter plane.

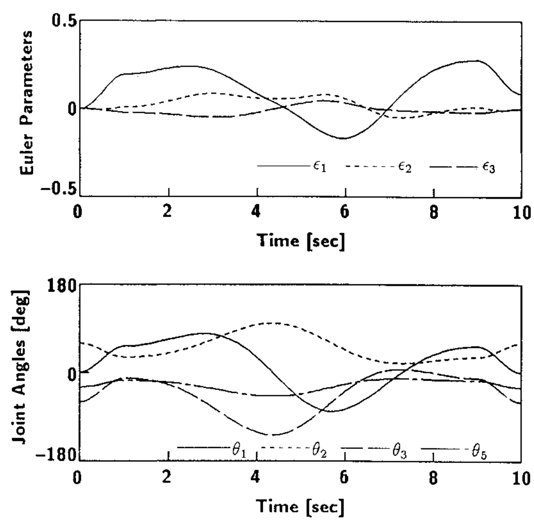

Fig. 6 Attitude and joint angle motion ($\sigma = \sigma_{\max}$).

The angle ϕ of $\sigma_{\max}$ is accidentally the same as that of $\sigma_{\min}$ in this case. Figure 4 shows a section of Fig. 3 when cut by a plane including the points $\sigma_{\max}$ and $\sigma_{\min}$ ($\phi = 90$ deg and $\phi = 270$ deg). From Figs. 3 and 4, σ varies rapidly in the vicinity of $\sigma = \sigma_{\min}$. This means that the attitude change is easily induced in a limited direction.

Next is a simulation study of the attitude change. The arm path in the $p_1 p_2$ plane is shown in Fig. 5. In this figure, the path starts from the origin (A) and moves along the unit circle ($B \to C \to D \to E \to F$) and finally returns to the origin (G). Here the unit vector $\boldsymbol{e}$ is set in the direction of $\sigma = \sigma_{\max}$ and $\sigma = \sigma_{\min}$ and the rotation angle ψ is 10 deg. Table 2 shows the values of $\boldsymbol{a}$ and $\boldsymbol{b}$ for the approximate solutions and the modified ones. In both cases, the modified solutions are close to the approximate solutions. Only 2 iterations are necessary for the modified solutions to converge from the approximate solutions. Table 3 shows the final attitude change Δ_ϵ in both cases. To compare $\sigma = \sigma_{\max}$ with $\sigma = \sigma_{\min}$, the orthogonal matrix $\boldsymbol{U}$ using ϕ and ϑ is defined in each case as

$$\boldsymbol{U} = \begin{bmatrix} \cos\phi\cos\vartheta & \sin\phi\cos\vartheta & \sin\vartheta \\ -\sin\phi & \cos\phi & 0 \\ -\cos\phi\sin\vartheta & -\sin\phi\sin\vartheta & \cos\vartheta \end{bmatrix} \tag{35}$$

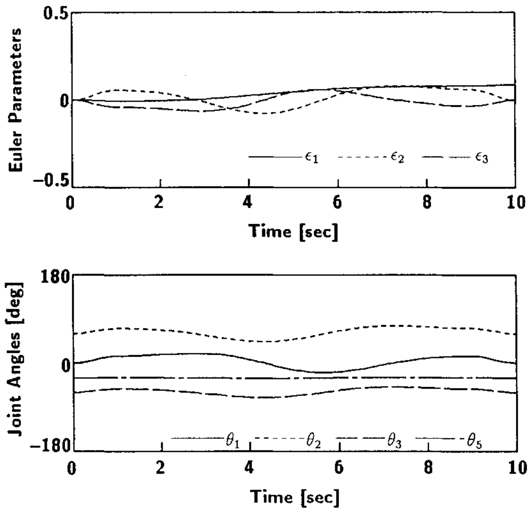

Fig. 7 Attitude and joint angle motion ($\sigma = \sigma_{\min}$).

Table 3 shows the components of $U\Delta_\epsilon$, whose desired values are $[0.0872, 0, 0]^T$. For $\sigma = \sigma_{\max}$, the attitude error of the approximate solution is large because of the large arm motion. Even so, the modified solution causes the desired attitude change.

Figures 6 and 7 show the attitude and the joint angles for large amplitudes ($\theta_1, \theta_2, \theta_3$, and θ_5) of the modified solution with $\sigma = \sigma_{\max}$ and that with $\sigma = \sigma_{\min}$, respectively. To compare Figs. 6 and 7, the satellite attitude ϵ is expressed by the components of $U\epsilon$ ($= [\epsilon_1, \epsilon_2, \epsilon_3]^T$). These figures show a case where the parameters p_1 and p_2 move from A to B in 1 s, from B to F in 8 s, and from F to G in 1 s. However, the time has no essential meaning because the momentum and the angular momentum are conserved at zero. From these figures, the movements of the attitude and the joint angles of $\sigma = \sigma_{\max}$ are clearly much larger than those of $\sigma = \sigma_{\min}$.

Conclusion

The path-planning method of a space robot manipulator arm was investigated in this paper. The purpose of the path planning is to control the satellite's attitude by arm motion where the arm's final posture is the same as the initial one. An algorithm has been proposed to obtain an arm path that minimizes the joint angle movement. An attitude control measure that shows the difficulty of the attitude change for a given direction has also been introduced. Numerical studies have been executed using a space robot model with an arm of six degrees of freedom. The results show that there are some directions where the attitude change is easily achieved by the arm motion. The results also show that the proposed algorithm gives the desired attitude change.

If the arm motion becomes too large by the proposed method, a simple way of solving the problem is to increase the number of rotations of the arm. As the attitude change is in proportion to the area integration in the parameter plane, the amplitude of the arm motion is decreased almost in inverse proportion to the square root of the number of rotations.

This paper focuses on a case where the angular momentum is conserved at zero during the arm motion. When the angular momentum is kept at a nonzero value, not only the spatial path but also the time path of the arm must be considered. However, the approach in the paper is basically applicable to this, and the path-planning method is currently being developed.

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